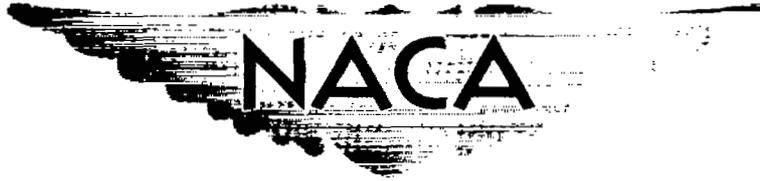


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RESEARCH MEMORANDUM

ANALYSIS OF TWO-STAGE COUNTERROTATING TURBINE
EFFICIENCIES IN TERMS OF WORK AND
SPEED REQUIREMENTS

By William T. Wintucky and Warner L. Stewart

Lewis Flight Propulsion Laboratory
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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WASHINGTON

March 18, 1958

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RESEARCH MEMORANDUM

ANALYSIS OF TWO-STAGE COUNTERROTATING TURBINE EFFICIENCIES

IN TERMS OF WORK AND SPEED REQUIREMENTS

By William T. Wintucky and Warner L. Stewart

SUMMARY

Two-stage counterrotating turbine efficiencies are analyzed in terms of mean-section, velocity-diagram parameters. Work and speed characteristics are combined in a basic parameter λ , which is defined as the ratio of the square of the average of the stage mean-section blade speed to the actual over-all specific work output. Efficiencies considered on this basis are total or aerodynamic, rating, and static. Range of λ from 0 to 0.375 is applicable in turbojet, turbopump, and accessory-drive turbines. Limits imposed are positive reaction and impulse across the first- or second-stage rotor or both. It is assumed that the flow is one-dimensional and that the blade specific losses are proportional to the average specific kinetic energy of the flow.

The turbine of this analysis has the same limits and general characteristics as a conventional two-stage turbine previously reported. The over-all efficiencies were slightly higher by 2 to 4 points, but within the assumptions of the analysis, were due to the elimination of the inter-stage stator. There was no significant change in the over-all efficiency for the first- to second-stage rotor speed ratios of 0.5, 1.0, and 2.0. Changing the rotor speed ratio from 1.0 did change the limiting value of λ with the value decreasing as the ratio decreased and increasing as the ratio increased. A major effect of changing the rotor speed ratio was to change the degree of the work split for maximum efficiencies. As the rotor speed ratio decreased, the first-stage to total-work ratio decreased, and an increased rotor speed ratio increased this work ratio at which maximum efficiencies occurred.

INTRODUCTION

As part of the general program concerned with the study of turbine performance characteristics, the NACA Lewis laboratory is currently conducting an analysis of the effect of work and speed requirements on turbine efficiency. The fundamental parameter used in this analysis is

the same as Parson's characteristic number described in reference 1, which is defined as the ratio of the square of the mean-section blade speed to the specific work output. References 2 and 3 involve the study of single-stage efficiency characteristics whereas reference 4 presents those for the conventional two-stage turbine.

This report represents an extension of the material presented in reference 4 to the case of the counterrotating turbine where the inter-stage stator is omitted and the second-stage rotor blade velocity is in the direction opposite to that of the first-stage rotor. This type of turbine is of interest in unusual turbojet and turbopump applications. Fundamental assumptions and limits used in reference 4 will also be used herein. Since both rotors are free to rotate at different blade speeds, the effect of variation in these speeds must be considered.

The three types of efficiency considered herein are the same as those studied in reference 4:

- (1) Total or aerodynamic efficiency, which includes all aerodynamic losses
- (2) Rating efficiency, which, in addition to the aerodynamic losses, considers the turbine exit whirl a loss (used in jet-engine analysis)
- (3) Static efficiency, which, in addition to the aerodynamic losses, considers the turbine exit total velocity head a loss (used in turbopump and accessory-drive turbine analyses)

In presenting the analysis results, the range of λ covered will include those encountered in the turbojet, turbopump, and auxiliary-drive fields. Results for the equal speed case, that is, where both rotors are operating at the same blade speed, will first be considered for zero and then varying negative turbine exit whirls. These results will then be compared with those presented in reference 4 for the conventional two-stage turbine. Finally, the effect of varying the rotor speed ratio on the efficiency characteristics and power limitations will be described.

SYMBOLS

- A turbine blade surface area, sq ft
- B parameter equal to $K \frac{A}{w}$
- C parameter describing velocity-diagram type for stage total-efficiency equation

E	specific kinetic-energy level, Btu/lb
F	ratio of stage to average blade speed
g	acceleration due to gravity, 32.17 ft/sec ²
h	specific enthalpy, Btu/lb
$\Delta h'$	specific work output, Btu/lb
J	mechanical equivalent of heat, 778.2 ft-lb/Btu
K	constant or proportionality
L	loss in kinetic energy, Btu/lb
U	mean-section blade speed, ft/sec
V	absolute gas velocity, ft/sec
ΔV_u	change in tangential whirl across the rotors
W	relative gas velocity, ft/sec
w	weight-flow rate, lb/sec
η	total efficiency, based on total-pressure ratio across turbine
η_s	static efficiency, based on ratio of total to static pressure across turbine
η_x	rating efficiency, based on total pressure upstream of turbine and pressure downstream of turbine equal to sum of static pressure and axial component of velocity head
λ	work-speed parameter, $U^2/gJ\Delta h'$

Subscripts:

a	first stage
av	average
b	second stage
id	ideal
R	rotor

S	stator
ST	stage
u	tangential component
x	axial component
0	upstream of turbine
1	station between first-stage stator and rotor
2,3	station between first and second stage
4	downstream of turbine

Superscripts:

'	absolute total state
-	over-all

METHOD OF ANALYSIS

The INTRODUCTION pointed out that the fundamental parameter used in determining the counterrotating turbine efficiency characteristics is λ . This parameter is defined herein as

$$\lambda = \frac{U_{av}^2}{gJ\Delta h'} \quad (1)$$

where $\Delta h'$ is the over-all specific work output, and U_{av} is the average of the two rotor mean-section blade speeds. That is

$$U_{av} = \frac{U_a + U_b}{2} \quad (2)$$

In providing for the variation in the ratio of stage blade speeds, this report will use the speed ratio defined by

$$\frac{F_a}{F_b} = \frac{U_a}{U_b}$$

that is, the ratio of first- to second-stage speed. By definition

$$F_a = \frac{U_a}{U_{av}} \quad (3a)$$

$$F_b = \frac{U_b}{U_{av}} \quad (3b)$$

In the analysis, a range of F_a/F_b will be selected. It will then be desired to obtain F_a and F_b for use in the efficiency calculations. The equations to do this can be obtained by the proper combination of equations (3a) and (3b) with equation (2) to yield

$$F_a = \frac{2}{\left(\frac{F_a}{F_b}\right)^{-1} + 1} \quad (4a)$$

$$F_b = \frac{2}{\frac{F_a}{F_b} + 1} \quad (4b)$$

These speed relations will now be used with others defining the velocity-diagram characteristics to obtain the turbine-efficiency equations.

Efficiency Equations

The total, rating, and static efficiencies are similar to those of the two-stage analysis of reference 4 and are now developed to include the effects of the speed ratio.

Stage total efficiency. - The stage total efficiency is defined to include all aerodynamic losses as

$$\eta_{ST} = \frac{\Delta h_{ST}^i}{\Delta h_{id,ST}^i} \quad (5)$$

where Δh_{ST}^i is the actual specific work output, and $\Delta h_{id,ST}^i$ is the ideal specific work output corresponding to the total-pressure ratio across the turbine stage. This efficiency is related to the work-speed and velocity-diagram parameters as

$$\eta_{ST} = \frac{\lambda_{ST}}{\lambda_{ST} + BC_{ST}} \quad (6a)$$

where, as in the two-stage analysis (ref. 4)

$$\lambda_{ST} = \frac{U_{ST}}{gJ\Delta h_{ST}^i} \quad (6b)$$

and

$$B = K \frac{A}{w} \quad (6b)$$

The equation for C is originally derived in reference 2 and is modified for a two-stage turbine in reference 4. For the first stage, C_a has the same velocity relation.

$$C_a = 4\lambda_a \frac{(v_x^2)_{av}}{gJ\Delta h'_a} + \left(\frac{v_{u,1}}{\Delta v_{u,a}}\right)^2 + \left(\frac{v_{u,1}}{\Delta v_{u,a}} - \lambda_a\right)^2 + \left(\frac{v_{u,1}}{\Delta v_{u,a}} - \lambda_a - 1\right)^2 \quad (6c)$$

The second stage C_b is shortened by two expressions since the interstage stator is eliminated. Also the whirl parameter is changed to terms of the turbine exit-whirl parameter $v_{u,4}/\Delta v_{u,b}$ by the following relation:

$$\frac{v_{u,3}}{\Delta v_{u,b}} = 1 + \frac{v_{u,4}}{\Delta v_{u,b}}$$

Now the expression for C_b is

$$C_b = 2\lambda_b \frac{(v_x^2)_{av}}{gJ\Delta h'_b} + \left(\frac{v_{u,4}}{\Delta v_{u,b}} - \lambda_b + 1\right)^2 + \left(\frac{v_{u,4}}{\Delta v_{u,b}} + \lambda_b\right)^2 \quad (6d)$$

Velocity diagrams and station nomenclature are shown in figure 1. For ease of comparison with the conventional two-stage turbine, the direction of rotor rotation is positive.

The assumptions involved in the derivation of C are as follows since C is not derived here:

(1) The stage specific energy loss $\Delta h'_{id,ST} - \Delta h'_{ST}$ is assumed equal to the sum of the stator and rotor specific flow losses $I_S + I_R$. These flow losses, expressed in units of Btu per pound, are defined as the difference between the ideal and actual specific kinetic energy obtained through expansion to the blade exit static pressure. The validity of this assumption is investigated in appendix C of reference 2, where it is shown to be correct except for the small effect of absolute enthalpy variations through the stage.

(2) The stage specific flow loss $I_S + I_R$ is assumed proportional to the surface area per unit weight flow and the specific kinetic-energy level of the flow; that is,

$$L_S + L_R \propto \frac{A}{W} E \quad (7)$$

This assumption can be shown to be valid from boundary-layer considerations.

(3) The specific kinetic-energy level E of the flow is assumed to be representable by the average of the specific kinetic energies entering and leaving the blade rows where the velocities involved are relative to the blade rows.

The assumed constants are kept the same as in the previous analyses; $B = 0.030$ and $(v_x^2)_{av}/gJ\Delta h' = 0.245$. Equations (6c) and (6d) as presented include a stage specific work term. This stage specific work is modified so that the assumed constant with the over-all specific work $(v_x^2)_{av}/gJ\Delta h'$ may be used.

$$\left. \begin{aligned} \frac{(v_x)_{av}}{gJ\Delta h'_{ST}} &= \frac{(v_x)_{av}}{gJ\Delta h'} \times \frac{U_{av}^2}{gJ\Delta h'_{ST}} \\ &= \frac{(v_x)_{av}}{gJ\Delta h'} \times \frac{1}{\lambda} \frac{U_{av}^2}{U_{ST}^2} \frac{U_{ST}^2}{gJ\Delta h'_{ST}} \\ &= \frac{\lambda_{ST}}{\lambda_{FST}^2} \frac{(v_x^2)_{av}}{gJ\Delta h'} \end{aligned} \right\} \quad (8)$$

Using equations (6b) and (6c) or (6d) in equation (6a) results in an equation for stage efficiency.

Over-all total efficiency. - The over-all total efficiency including all aerodynamic losses is defined as:

$$\bar{\eta} = \frac{\Delta h'}{\Delta h'_{id}} \quad (9)$$

where $\Delta h'$ is the actual specific work output across the turbine, and $\Delta h'_{id}$ is the ideal specific work output corresponding to the total-pressure ratio across the turbine. The effect of turbine interstage re-heat is not considered since an enthalpy level would have to be specified.

Equation (9) can be written as

$$\bar{\eta} = \frac{\Delta h'_a + \Delta h'_b}{\Delta h'_{id,a} + \Delta h'_{id,b}} \quad (10)$$

Multiplying the numerator and denominator of equation (10) by

$$\frac{\Delta h'_a}{\Delta h'_{id,a}} \frac{\Delta h'_b}{\Delta h'_{id,b}} \frac{U_{av}^2}{gJ\Delta h'_a \Delta h'_b}$$

gives

$$\bar{\eta} = \frac{\frac{\Delta h'_a}{\Delta h'_{id,a}} \frac{\Delta h'_b}{\Delta h'_{id,b}} \left(\frac{U_{av}^2}{gJ\Delta h'_a} + \frac{U_{av}^2}{gJ\Delta h'_b} \right)}{\frac{\Delta h'_a}{\Delta h'_{id,a}} \frac{U_{av}^2}{gJ\Delta h'_a} + \frac{\Delta h'_b}{\Delta h'_{id,b}} \frac{U_{av}^2}{gJ\Delta h'_b}}$$

Substituting the relations of η_a , η_b , λ_a , λ , F_a , and F_b , results in the following expression for over-all total efficiency:

$$\bar{\eta} = \frac{\eta_a \eta_b \left[\lambda_a + \lambda_b \left(\frac{F_a}{F_b} \right)^2 \right]}{\eta_a \lambda_a + \eta_b \lambda_b \left(\frac{F_a}{F_b} \right)^2} \quad (11)$$

Over-all rating efficiency. - The kinetic energy in jet-engine turbine exit whirl is lost since it does not add to thrust. Single-stage turbine rating efficiency was derived in reference 2 and modified for the two-stage turbine in reference 4 to

$$\bar{\eta}_x = \frac{\bar{\eta}}{1 + \frac{v_{u,4}^2}{2gJ\Delta h'_{id}}} \quad (12)$$

Making appropriate substitutions in equation (12)

$$\bar{\eta}_x = \frac{\bar{\eta}}{1 + \frac{1}{2} \frac{v_{u,4}^2}{\Delta v_{u,b}^2} \frac{\Delta v_{u,b}^2}{U_b^2} \frac{U_b^2}{U_{av}^2} \frac{U_{av}^2}{gJ\Delta h'} \frac{\Delta h'}{\Delta h'_{id}}}$$

Using the definitions of λ_b , λ , F_b , and $\bar{\eta}$, results in the following expression for rating efficiency:

$$\bar{\eta}_x = \frac{\bar{\eta}}{1 + \frac{\eta \lambda F_b}{2\lambda_b^2} \left(\frac{V_{u,4}}{\Delta V_{u,b}} \right)^2} \quad (13)$$

Over-all static efficiency. - In applications such as turbopump and accessory drives, the entire velocity head is a loss chargeable to the turbine. For the single-stage turbine this efficiency was also derived in reference 2 and modified for the two-stage turbine in reference 4 to

$$\bar{\eta}_s = \frac{\bar{\eta}}{1 + \frac{V_{x,4}^2 + V_{u,4}^2}{2gJ\Delta h'_{id}}} \quad (14)$$

As in the other analyses it is assumed that $V_{x,4} = (V_x^2)_{av}$ and with the proper substitutions made in equation (14)

$$\bar{\eta}_s = \frac{\bar{\eta}}{1 + \frac{1}{2} \frac{\Delta h'}{\Delta h'_{id}} \left[\frac{(V_x^2)_{av}}{gJ\Delta h'} + \frac{V_{u,4}^2}{\Delta V_{u,b}^2} \times \frac{\Delta V_{u,b}^2}{U_b^2} \times \frac{U_b^2}{U_{av}^2} \times \frac{U_{av}^2}{gJ\Delta h'} \right]}$$

Again using the definitions of $\bar{\eta}$, λ_b , F_b , and λ , results in an expression for static efficiency

$$\bar{\eta}_s = \frac{\bar{\eta}}{1 + \frac{\bar{\eta}}{2} \left[\frac{(V_x^2)_{av}}{gJ\Delta h'} + \frac{\lambda F_b^2}{\lambda_b^2} \left(\frac{V_{u,4}}{\Delta V_{u,b}} \right)^2 \right]} \quad (15)$$

Work-Speed Parameter Relation

The sum of the work done by both stages is equal to the total work done by the turbine.

$$\Delta h' = \Delta h'_a + \Delta h'_b$$

Multiplying by gJ and dividing by U_{av}^2 results in

$$\frac{gJ\Delta h'}{U_{av}^2} = \frac{gJ\Delta h'_a}{U_a^2} \left(\frac{U_a}{U_{av}}\right)^2 + \frac{gJ\Delta h'_b}{U_b^2} \left(\frac{U_b}{U_{av}}\right)^2$$

When the definitions of λ , λ_a , λ_b , F_a , and F_b are substituted, the following relation results:

$$\frac{1}{\lambda} = \frac{F_a^2}{\lambda_a} + \frac{F_b^2}{\lambda_b} \quad (16)$$

First-Stage-Stator Exit-Whirl Considerations

The last parameter needed for calculation of efficiencies is the first-stage-stator exit-whirl parameter ($V_{u,1}/\Delta V_{u,a}$). This was one of the independent variables in reference 4, but with the elimination of the interstage stator, it becomes a dependent variable in this analysis. It is developed from the tangential whirl relation across both rotors.

$$V_{u,1} - V_{u,2} = \Delta V_{u,a}$$

$$V_{u,3} - V_{u,4} = \Delta V_{u,b}$$

The absolute tangential flow velocity out of the first rotor is the same as that going into the second-stage rotor but opposite in sign ($V_{u,2} = -V_{u,3}$). By combining the expressions of stage tangential velocities and dividing by $\Delta V_{u,a}$, the following is obtained:

$$\frac{V_{u,1}}{\Delta V_{u,a}} - 1 = \frac{-\Delta V_{u,b}}{\Delta V_{u,a}} - \frac{V_{u,4}}{\Delta V_{u,a}}$$

$$\frac{V_{u,1}}{\Delta V_{u,a}} = 1 - \frac{\Delta V_{u,b}}{U_b} \left(\frac{U_b}{U_{av}}\right) \left(\frac{U_{av}}{U_a}\right) \frac{U_a}{\Delta V_{u,a}} \left(1 - \frac{V_{u,4}}{\Delta V_{u,b}}\right)$$

Using relations of λ_a , λ_b , F_a , and F_b

$$\frac{V_{u,1}}{\Delta V_{u,a}} = 1 - \left(\frac{\lambda_a}{\lambda_b}\right) \left(\frac{F_b}{F_a}\right) \left(1 + \frac{V_{u,4}}{\Delta V_{u,b}}\right) \quad (17)$$

Method of Calculations and Limits

The four parameters required for solution of the over-all efficiency equations are F_a/F_b , $V_{u,4}/\Delta V_{u,b}$, λ , and λ_b . These parameters will be selected to cover the range considered of interest. Once these parameters have been selected, the over-all efficiencies can be calculated for a given set of conditions. Listed in table I are the equation relations used and the quantities needed for their calculations. The ranges and limits placed on the independent variables are now discussed.

Speed ratio F_a/F_b . - Since the speed ratio is one of the major independent variables of this analysis, F_a/F_b ratios of 0.5, 1.0, and 2.0 are covered to get an effect of ratio variation. However, the primary emphasis of the analysis is placed on the equal speeds case in order to make a comparison with the conventional two-stage analysis.

Turbine exit-whirl parameter $V_{u,4}/\Delta V_{u,b}$. - The range of this variable was limited to zero and negative values, since, for a given work and speed combination, maximum efficiencies do not occur with positive exit whirls. The upper limit was selected as zero and the lower limit as -0.4. Using negative turbine exit whirls, allows a lower λ to be used within the limits of the analysis as will be shown. The range of this parameter is then

$$-0.4 \leq \frac{V_{u,4}}{\Delta V_{u,b}} \leq 0 \quad (18)$$

Over-all work-speed parameter λ . - The upper value of λ for which calculations were made was arbitrarily selected as 0.375. This value adequately determines the over-all efficiency levels and the flatness of the upper portion of the efficiency curves. Also this point was used in the two-stage analysis (ref. 4), and a direct comparison can be made.

The lower limit of λ is based on impulse conditions across both rotor blade rows. Reaction across any blade row is limited to positive or zero, since it has been found that blade losses markedly increase with negative reaction. The value of the loss parameter B is more valid for positive and zero reaction than negative reaction. For ease of analysis, the assumption is made that the axial component of velocity through the turbine is constant. The specification of impulse conditions and constant axial velocity across the rotors means that the relative whirl velocities entering and leaving the rotors are equal but opposite in sign

$$W_{u,1} = -W_{u,2}$$

$$W_{u,3} = -W_{u,4}$$

since

$$\Delta V_{u,a} = W_{u,1} - W_{u,2}$$

$$\Delta V_{u,b} = W_{u,3} - W_{u,4}$$

By rearranging and using velocity-diagram considerations, equations for impulse conditions across both rotors are developed.

$$\frac{V_{u,1}}{\Delta V_{u,a}} = \lambda_a + \frac{1}{2} \quad (19)$$

$$\frac{V_{u,4}}{\Delta V_{u,b}} = \lambda_b - \frac{1}{2} \quad (20)$$

One-point operation and, therefore, the lowest value of λ occurs when both rotors are at impulse conditions. To get an expression, for this point, equations (4a), (16), (19), and (20) are substituted in equation (17). The resultant equation for λ in terms of F_a/F_b and $V_{u,4}/\Delta V_{u,b}$ is

$$\lambda = \frac{\left(\frac{F_a}{F_b} + 1\right)^2 \left(\frac{1}{2} + \frac{V_{u,4}}{\Delta V_{u,b}}\right)}{8 \left\{ \frac{1}{2} + \frac{F_a}{F_b} \left[1 + \frac{V_{u,4}}{\Delta V_{u,b}} + \frac{F_a}{F_b} \left(\frac{1}{2} + \frac{V_{u,4}}{\Delta V_{u,b}} \right) \right] \right\}} \quad (21)$$

The graphical representation of this equation is shown in figure 2.

Second-stage work-speed parameter λ_b . - The last independent variable is the second-stage work-speed parameter which, after F_a/F_b , $V_{u,4}/\Delta V_{u,b}$, and λ are selected, determines the work split of the two stages. The upper limit on λ_b occurs when the first-stage rotor is at impulse conditions. So using equations (16) and (19) in equation (17) gives the following equation for the upper limit of λ_b :

$$\lambda_b = \frac{\left(\frac{F_b}{F_a}\right) \left(\frac{F_b}{2F_a} + 1 + \frac{V_{u,4}}{\Delta V_{u,b}}\right)}{\frac{1}{2\lambda F_a^2} - 1} \quad (22)$$

The lower limit of λ_b occurs when the second-stage rotor is at impulse conditions. This is obtained from equation (20) for a specified turbine exit whirl

$$\lambda_b = \frac{1}{2} + \frac{V_{u,4}}{\Delta V_{u,b}} \quad (23)$$

RESULTS OF ANALYSIS

In presenting the results of the counterrotating turbine analysis, equal rotor speeds are first considered with zero turbine exit whirl. The effect of adding exit whirl on turbine efficiencies is then shown. A comparison is made with the two-stage turbine analysis of reference 4 to show possible gains in efficiency due to the elimination of the inter-stage stator. Finally the effect of varying the rotor speed ratio on over-all efficiency and work split is presented for first- to second-stage rotor speed ratios of 0.5 and 2.0.

Equal Rotor Speed

Zero turbine exit whirl. - Over-all total and static turbine efficiencies are presented in figure 3 for various over-all λ . For the zero turbine exit-whirl case, the total and rating efficiencies are the same. The over-all efficiencies are presented as a function of the first-stage-stator exit-whirl parameter $V_{u,1}/\Delta V_{u,a}$ over a range of values from 0 to 0.8. At specified values of λ , a range of λ_b is covered to indicate the range of maximum efficiencies within the impulse limits of the analysis. The region of maximum efficiencies occurs over a range of $V_{u,1}/\Delta V_{u,a}$ of approximately 0.4 to 0.65.

At a λ of 0.375, the efficiency curve is flat over a wide range of first-stage-stator exit whirls. The maximum over-all efficiency from figure 3 for $\lambda = 0.374$ occurs at a λ_b of 1.2, which is very conservative. The first stage λ_a equals 0.65, which is the equivalent of a single-stage turbine with zero exit whirl. At a higher λ , the λ_b at which maximum efficiency occurred would be higher and therefore more conservative. This is considered a questionable design region since a single-stage turbine could now do the work with comparable efficiencies.

As λ is reduced, the range of operation within the limits of this analysis is reduced until at impulse conditions across both rotors, a 1-point operation is reached at $\lambda = 0.125$ for zero turbine exit whirl. The dashed curve represents impulse conditions across the second-stage rotor, and the dash-dot curve is impulse across the first-stage rotor. The stage impulse limits are not shown for the higher λ 's since the over-all efficiencies for these points would be much lower than the peak values and could fall off the graph. Maximum total- and static-efficiency points for a given λ occur at the same combinations of λ , λ_b , and $V_{u,1}/\Delta V_{u,a}$. It is interesting to note that for zero exit whirl, the first-stage-stator whirl parameter $V_{u,1}/\Delta V_{u,a}$ is equal to one minus the ratio of the stage works $\left(1 - \frac{\Delta h_b^i}{\Delta h_a^i}\right)$.

In reference 4, figures 3 and 4 are the total and static efficiencies for the conventional two-stage turbine. The dashed line represents the interstage stator at impulse conditions, which is the equivalent of this analysis. Comparing this condition with this analysis, shows that the curves of both analyses are similar in shape and occur at approximately the same values of $V_{u,1}/\Delta V_{u,a}$.

Effect of turbine exit whirl. - The general effect of adding negative turbine exit whirl, as discussed in the section on λ limits, is to reduce the λ at which impulse conditions are reached across both rotors. Turbine exit whirl is not considered for λ above 0.25 since the turbine is then definitely operating at maximum over-all efficiencies without exit whirl.

Figures 4(a), 4(b), and 4(c) are over-all total, rating, and static efficiencies over a range of turbine exit whirls from 0 to -0.3 presented on the same basis as the zero turbine exit-whirl case. Efficiency values for turbine exit whirl of -0.4 are not shown since they are much lower than the other exit whirls, and some points do not fall in the range of efficiencies presented. The peak efficiency points for $V_{u,4}/\Delta V_{u,b} = -0.4$ are presented in table II. Points were computed to determine the shape of the curves but only the even valued λ_b points are presented on the curves.

The efficiency curves are fairly flat such that design diagram parameters could be varied without effecting the over-all efficiencies to any great extent. For example, at a λ of 0.20, the first-stage-stator exit-whirl parameter could vary from 0.3 to 0.6 without changing the total efficiency more than 1 point or the rating and static efficiencies more than 2 points. The range of design conditions for a given λ is somewhat extended by the addition of turbine exit whirl since the lower limit on λ_b is reduced.

Table II is a compilation of calculated points of maximum efficiencies. At increased turbine exit whirls, maximum total efficiencies can occur at values of $V_{u,1}/\Delta V_{u,a}$ different from those of the rating and static efficiencies. A complete set of data is presented for total, rating, and static efficiencies. Work splits are presented only for the rating and static efficiencies since these methods of rating the turbine are of prime interest. Total efficiency is not affected to any great extent by the amount of turbine exit whirl since it is only a measure of aerodynamic performance.

Figure 5 is a plot of the peak efficiency points over the range of λ considered. For clarity on figure 5 the total-efficiency curve does not have all of the exit-whirl points from table II plotted on it. From the rating- and static-efficiency curves it can be seen that the use of

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turbine exit whirl can increase the over-all efficiencies for low values of λ . This is due to the fact that the energy lost in the small amount of whirl added is more than offset by the improvement in velocity diagrams. The efficiencies drop off as λ is reduced because of increased viscous losses and go to zero as λ is reduced to zero.

Work split. - Figure 6(a) shows the work split of the two stages as a ratio of first-stage work to total work $\Delta h'_a/\Delta h'$ for the peak efficiency points presented in figure 5. This is expressed in terms of the work-speed parameters, and using the relations of λ , λ_a , and F_a , the work split is developed as

$$\frac{\Delta h'_a}{\Delta h'} = \frac{\frac{U_{av}^2}{gJ\Delta h'}}{\frac{U_a^2}{gJ\Delta h'} \times \frac{U_{av}^2}{U_a^2}} = \frac{\lambda F_a^2}{\lambda_a} \quad (24)$$

The solid line represents the zero exit-whirl case. As exit whirl is added, the second stage is permitted to do more work. The dashed-dot line is drawn through points of impulse across both rotors at specified exit whirls. These points lie in a straight line which reduces $\Delta h'_a/\Delta h'$ to 0.5 when λ goes to zero. Maximum over-all rating and static efficiencies occur in the region of $\Delta h'_a/\Delta h' = 0.65$. There is some scatter in the points since the points are computed from specified points and are not necessarily points of absolute maximum efficiency. The work-split characteristics of figure 6(a) compared with figure 9 of reference 4 are the same below a λ of 0.025.

Comparison with two-stage efficiency results. - The counterrotating turbine is now compared with the conventional two-stage turbine of reference 4 by superimposing the curves of figure 5 with those of reference 4 and thus presenting figure 6. Within the assumptions of this analysis, by elimination of the interstage stator, the total losses are less thereby increasing the total, rating, and static efficiencies. At $\lambda = 0.375$, the counterrotating turbine over-all efficiencies are about 2 points higher than the two-stage turbine. As λ reduces to 0.25, the counterrotating turbine over-all efficiencies are about 3 points higher than reference 4. This difference slightly increases to about 4 points as λ is reduced.

The absolute level of the efficiencies may be somewhat higher than that encountered in actual practice since the loss coefficient B is for a single-stage turbine. It is not known to what extent this coefficient would actually vary in this analysis. Variations in the assumed constants and assumptions were studied in the conventional two-stage analysis of reference 4.

Varying Speed Ratio

One effect of changing the rotor speed ratio is to change the λ at which impulse conditions are reached for a specified turbine exit whirl. It was previously mentioned that the graphical representation of the impulse limit over a range of speed ratios and turbine exit whirls is shown in figure 2. For zero turbine exit whirl the impulse limit is unaffected by the speed ratio. As F_a/F_b increases above 1, the impulse limit for all negative whirls approaches the zero whirl value. When the F_a/F_b ratio decreases from 1 to 0, the impulse limit decreases to a limiting value for each turbine exit whirl.

Effect of speed ratio on over-all efficiency. - As was pointed out in the method of calculations, two reasonably different speed ratios of 0.5 and 2.0 are analyzed to determine the effects of varying speed ratios. The two ratios were completely analyzed in the same manner as the case of equal rotor speeds, and the results are tabulated in table II. From these tables, maximum over-all efficiency points for the various turbine exit whirls over the same λ range are plotted in figure 8. A comparison of figure 8 with figure 5 for all the efficiencies shows that in the flat portions of the curves there is no difference in efficiencies for the three ratios. In the λ region where the efficiency starts to drop off, there is about a 1-point difference for any efficiency, and this difference increases very slightly thereafter. From this it can be concluded that varying the speed ratio does not effect to any significant extent the level of efficiency for a given λ .

For the F_a/F_b ratio of 2.0 there is almost no change in over-all efficiency at a given λ for the range of turbine exit whirls considered. This is due to the fact that the first-stage rotor is operating at twice the speed of the second, and the energy in the whirl coming out of the second rotor is only a small percentage of the total energy.

In the case of the F_a/F_b ratio of 0.5, varying the turbine exit whirl at a given λ has a pronounced effect on the over-all peak rating and static efficiencies. This is due to the fact that the turbine exit-whirl energy is now a large percentage of the total energy of the turbine.

Effect of speed ratio on work split. - Shown in figures 6(b) and 6(c) are the first-stage to total-work ratios corresponding to the peak efficiency points plotted in figure 8. It can be seen that when the three speed ratios are compared, the major effect of varying F_a/F_b is to change significantly the general level of work splits $\Delta h_d/\Delta h'$ for which maximum over-all efficiencies occur.

The three figures (6(a), 6(b), and 6(c)) of work splits have the same general characteristics. For zero turbine exit whirl as λ is

reduced the first-stage work percentage increases and peaks at a λ of 0.125. From this point as λ is reduced the first-stage to total-work ratio is reduced as the impulse limit on both rotors describes a straight line (eq. (21)). Below these two curves is a region where the work split can be varied by adding turbine exit whirl without affecting the over-all total efficiency to any great extent. At maximum over-all efficiencies, first-stage to total work splits of 0.4 to 0.5, 0.6 to 0.7, and 0.8 to 0.9 occurred at the speed ratios of 0.5, 1.0, and 2.0, respectively.

SUMMARY OF RESULTS

An analysis of two-stage counterrotating turbine efficiencies over a range of work and speed requirements has been presented. The analysis was conducted using the same assumptions and limits as the two-stage and single-stage analyses previously reported. Total, rating, and static efficiencies are presented as a function of the first-stage-stator exit whirl with the over-all work-speed parameter, second-stage work-speed parameter, and turbine exit-whirl parameter as major variables. The efficiencies also are presented as peak-point curves and are compared with the conventional two-stage turbine efficiencies. The speed ratio is varied and the results compared. A summary of the pertinent results is as follows:

1. The counterrotating two-stage turbine with equal rotor speeds had the same general performance characteristics as the conventional two-stage turbine. The over-all limits are the same with the counterrotating turbine being more limited because of the absence of an interstage stator. The over-all efficiencies are from about 2 to 4 points higher than the conventional turbine depending on the region of λ . Within the assumptions of the analysis, this was due to the elimination of the interstage stator and its associated losses.

2. At speed ratios of 0.5, 1.0, and 2.0 there was no significant effect on the general level of over-all efficiency. In the λ region where the efficiencies started to drop off there was a slight variation of 1 to 2 points. Changing the speed ratio did change the λ at which impulse conditions were reached for a specified negative turbine exit whirl. Increasing the speed ratio increased the limiting λ and decreasing the speed ratio decreased the limiting λ . At a speed ratio of 2.0 for a specified λ there was no marked effect on over-all efficiency due to changing the turbine exit whirl because the energy in the second-stage whirl is a small percentage of the total energy. With a speed ratio of 0.5 for a given λ there was a definite effect on over-all rating and static efficiencies for the various turbine exit whirls. A major effect of changing the rotor speed ratio was to change the level at which the work split occurred for the peak over-all efficiency points. The first-stage work to total work splits for the speed ratios of 0.5, 1.0, and

2.0 occurred at levels of about 0.4 to 0.5, 0.6 to 0.7, and 0.8 to 0.9, respectively, for maximum over-all efficiency.

CONCLUDING REMARKS

This is a comparative analysis and the absolute level of the efficiencies presented herein may be somewhat higher than that encountered in actual practice. The purpose of the report is to show the effect on over-all efficiency levels for variations in work and speed requirements for a two-stage counterrotating turbine using the same constants and assumptions as in the analysis of the conventional two-stage turbine. Variations in the constants and assumptions were studied in the conventional two-stage analysis of reference 4 and were not considered here.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, December 7, 1957

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2. Stewart, Warner L.: Analytical Investigation of Single-Stage-Turbine Efficiency Characteristics in Terms of Work and Speed Requirements. NACA RM E56G31, 1956.
3. Wintucky, William T., and Stewart, Warner L.: Analysis of Efficiency Characteristics of a Single-Stage Turbine with Downstream Stators in Terms of Work and Speed Requirements. NACA RM E56J19, 1957.
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TABLE I. - FUNCTIONAL RELATIONS AND CALCULATION PROCEDURE
USED IN OBTAINING COUNTERROTATING TURBINE EFFICIENCIES

(a) Functional relations

Equation	Functional relation
(4a)	$F_a = f(F_a/F_b)$
(4b)	$F_b = f(F_a/F_b)$
(6a)	$\eta_a = f(\lambda, \lambda_a, F_a, V_{u,1}/\Delta V_{u,a})$
(6a)	$\eta_b = f(\lambda, \lambda_b, F_b, V_{u,4}/\Delta V_{u,b})$
(11)	$\bar{\eta} = f(\lambda_a, \lambda_b, \eta_a, \eta_b, F_a/F_b)$
(13)	$\bar{\eta}_x = f(\lambda, \lambda_b, \bar{\eta}, F_b, V_{u,4}/\Delta V_{u,b})$
(15)	$\bar{\eta}_s = f(\lambda, \lambda_b, F_b, \bar{\eta}, V_{u,4}/\Delta V_{u,b})$
(16)	$\lambda_a = f(\lambda, \lambda_b, F_a, F_b)$

(b) Calculation procedure

Quantity	Obtained from	Quantities needed
F_a/F_b	Range specified	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
λ	Range specified	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
λ_b	Range specified	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
$V_{u,4}/\Delta V_{u,b}$	Range specified	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
F_a	Eq. (4a)	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
F_b	Eq. (4b)	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
λ_a	Eq. (16)	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
$V_{u,1}/\Delta V_{u,a}$	Eq. (17)	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
η_a	Eq. (6a)	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
η_b	Eq. (6a)	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
$\bar{\eta}$	Eq. (11)	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
$\bar{\eta}_x$	Eq. (13)	○ ○ ○ ○ ○ ○ ○ ○ ○ ○
$\bar{\eta}_s$	Eq. (15)	○ ○ ○ ○ ○ ○ ○ ○ ○ ○

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TABLE II. - RESULTS OF MAXIMUM EFFICIENCY CALCULATIONS

(a) Speed ratio, F_a/F_b , 1

Turbine-exit-whirl parameter, $\frac{V_{u,4}}{\Delta V_{u,b}}$	Over-all work-speed parameter, λ	Maximum over-all total efficiency			Maximum over-all rating and static efficiencies, $\bar{\eta}_x$ and $\bar{\eta}_s$				
		λ_b	$\frac{V_{u,1}}{\Delta V_{u,a}}$	$\bar{\eta}$	λ_b	$\frac{V_{u,1}}{\Delta V_{u,a}}$	$\bar{\eta}_x$	$\bar{\eta}_s$	$\frac{\Delta h'_a}{\Delta h'}$
0	0.375	1.2	0.54	0.905	1.2	0.54	0.905	0.815	0.69
	.250	.7	.44	.903	.7	.44	.903	.813	.64
	.200	.6	.50	.897	.6	.50	.897	.808	.67
	.150	.5	.57	.879	.5	.57	.879	.793	.70
	.125	.5	.67	.848	.5	.67	.848	.768	.75
-0.1	0.250	0.7	0.50	0.904	0.7	0.50	0.901	0.812	0.64
	.200	.5	.40	.899	.5	.40	.896	.807	.60
	.150	.4	.46	.888	.4	.46	.884	.798	.62
	.111	.4	.65	.844	.4	.65	.841	.762	.72
-0.2	0.250	0.6	0.43	0.904	0.7	0.56	0.894	0.806	0.64
	.200	.5	.47	.900	.5	.47	.887	.800	.60
	.150	.4	.52	.888	.4	.52	.873	.789	.62
	.100	.3	.60	.850	.3	.60	.834	.751	.69
	.094	.3	.63	.836	.3	.63	.822	.747	.69
-0.3	0.250	0.6	0.50	0.904	0.7	0.61	0.882	0.796	0.64
	.200	.5	.53	.899	.6	.65	.871	.787	.67
	.150	.3	.30	.889	.4	.58	.853	.773	.62
	.100	.2	.30	.870	.3	.67	.808	.735	.67
	.071	.2	.61	.816	.2	.61	.766	.700	.64
-0.4	0.0427	0.1	0.57	0.767	0.1	0.57	0.610	0.567	0.58

TABLE II. - Continued. RESULTS OF MAXIMUM EFFICIENCY CALCULATIONS

(b) Speed ratio, F_a/F_b , 0.5

Turbine-exit-whirl parameter, $\frac{V_{u,4}}{\Delta V_{u,b}}$	Over-all work-speed parameter, λ	Maximum over-all total efficiency			Maximum over-all rating and static efficiencies, $\bar{\eta}_x$ and $\bar{\eta}_a$				
		λ_b	$\frac{V_{u,1}}{\Delta V_{u,a}}$	$\bar{\eta}$	λ_b	$\frac{V_{u,1}}{\Delta V_{u,a}}$	$\bar{\eta}_x$	$\bar{\eta}_s$	$\frac{\Delta h'_a}{\Delta h'}$
0	0.375	1.2	0.37	0.905	1.2	0.37	0.905	0.814	0.44
	.250	.8	.37	.903	.8	.37	.903	.813	.44
	.200	.6	.27	.898	.6	.27	.898	.809	.41
	.150	.5	.43	.880	.5	.43	.880	.794	.47
	.125	.5	.60	.833	.5	.60	.833	.756	.56
-0.1	0.250	0.7	0.22	0.903	0.8	0.44	0.897	0.808	0.44
	.200	.6	.34	.899	.6	.34	.893	.806	.40
	.150	.4	.10	.884	.4	.10	.877	.792	.33
	.125	.4	.44	.872	.4	.44	.868	.784	.44
	.107	.4	.59	.827	.4	.59	.823	.748	.52
-0.2	0.250	0.7	0.30	0.904	0.7	0.30	0.877	0.792	0.36
	.200	.6	.42	.899	.6	.42	.876	.791	.41
	.150	.4	.20	.891	.4	.20	.861	.778	.33
	.125	.4	.40	.872	.4	.40	.853	.772	.44
	.100	.3	.42	.864	.3	.42	.843	.763	.44
.086	.3	.58	.816	.3	.58	.794	.724	.49	
-0.3	0.250	0.7	0.39	0.903	0.7	0.39	0.844	0.765	0.36
	.200	.5	.14	.898	.6	.49	.846	.767	.41
	.150	.4	.30	.894	.5	.60	.832	.755	.47
	.125	.3	.00	.879	.4	.56	.825	.749	.44
	.100	.3	.49	.861	.3	.49	.816	.741	.41
	.080	.2	.14	.857	.2	.14	.783	.714	.29
.062	.2	.56	.794	.2	.56	.722	.664	.44	
-0.4	0.040	0.1	0.26	0.796	0.1	0.26	0.688	0.635	0.29
	.034	.1	.54	.726	.1	.54	.537	.504	.39

TABLE II. - Concluded. RESULTS OF MAXIMUM EFFICIENCY CALCULATIONS

(c) Speed ratio, F_a/F_b , 2.0

Turbine-exit-whirl parameter, $\frac{V_{u,4}}{\Delta V_{u,b}}$	Over-all work-speed parameter, λ	Maximum over-all total efficiency			Maximum over-all rating and static efficiencies, $\bar{\eta}_x$ and $\bar{\eta}_s$				
		λ_b	$\frac{V_{u,1}}{\Delta V_{u,a}}$	$\bar{\eta}$	λ_b	$\frac{V_{u,1}}{\Delta V_{u,a}}$	$\bar{\eta}_x$	$\bar{\eta}_s$	$\frac{\Delta h'_a}{\Delta h'}$
0	0.375	1.2	0.54	0.905	1.2	0.54	0.905	0.815	0.69
	.250	.7	.62	.904	.7	.62	.904	.814	.84
	.200	.5	.57	.900	.5	.57	.900	.810	.82
	.150	.5	.69	.882	.5	.69	.882	.796	.87
	.125	.5	.75	.862	.5	.75	.862	.779	.89
-0.1	0.250	0.6	0.59	0.904	0.6	0.59	0.903	0.813	0.81
	.200	.5	.61	.900	.5	.61	.899	.810	.82
	.150	.4	.64	.888	.4	.64	.887	.800	.83
	.125	.4	.71	.869	.4	.71	.869	.785	.86
	.115	.4	.74	.858	.4	.74	.857	.776	.87
-0.2	0.250	0.6	0.64	0.904	0.6	0.64	0.900	0.811	0.84
	.200	.4	.54	.900	.5	.65	.896	.807	.82
	.150	.3	.54	.892	.3	.54	.887	.800	.78
	.125	.3	.64	.879	.3	.64	.875	.791	.81
	.102	.3	.71	.854	.3	.71	.846	.767	.85
-0.3	0.250	0.5	0.60	0.904	0.6	0.68	0.895	0.807	0.81
	.200	.4	.60	.900	.4	.60	.890	.803	.78
	.150	.3	.60	.891	.3	.60	.881	.795	.78
	.125	.2	.46	.883	.2	.46	.870	.786	.72
	.100	.2	.60	.868	.2	.60	.859	.770	.78
	.083	.2	.68	.842	.2	.68	.813	.740	.82
-0.4	0.054	0.1	0.62	0.805	0.1	0.62	0.698	0.643	0.76

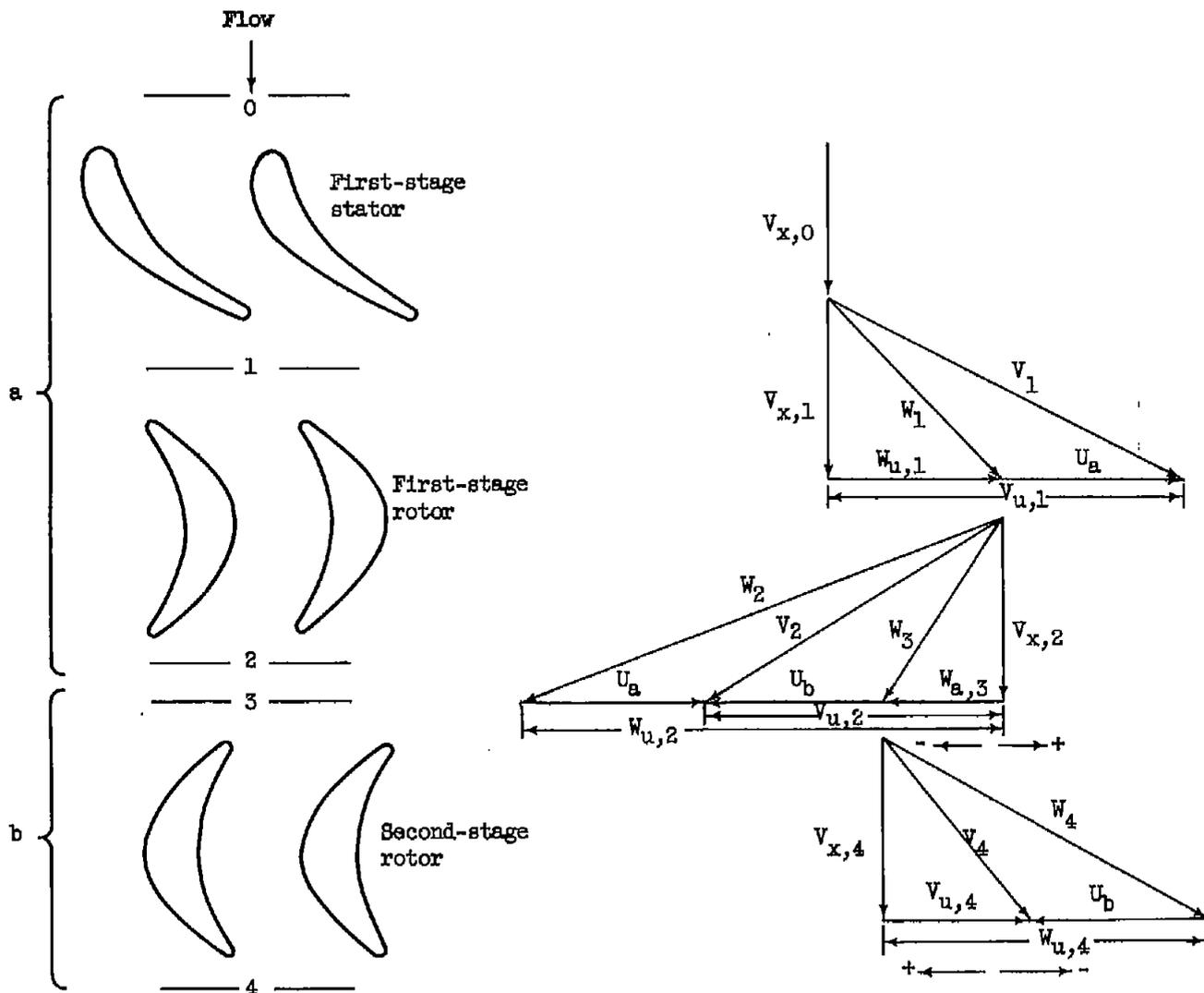


Figure 1. - Velocity diagrams and nomenclature.

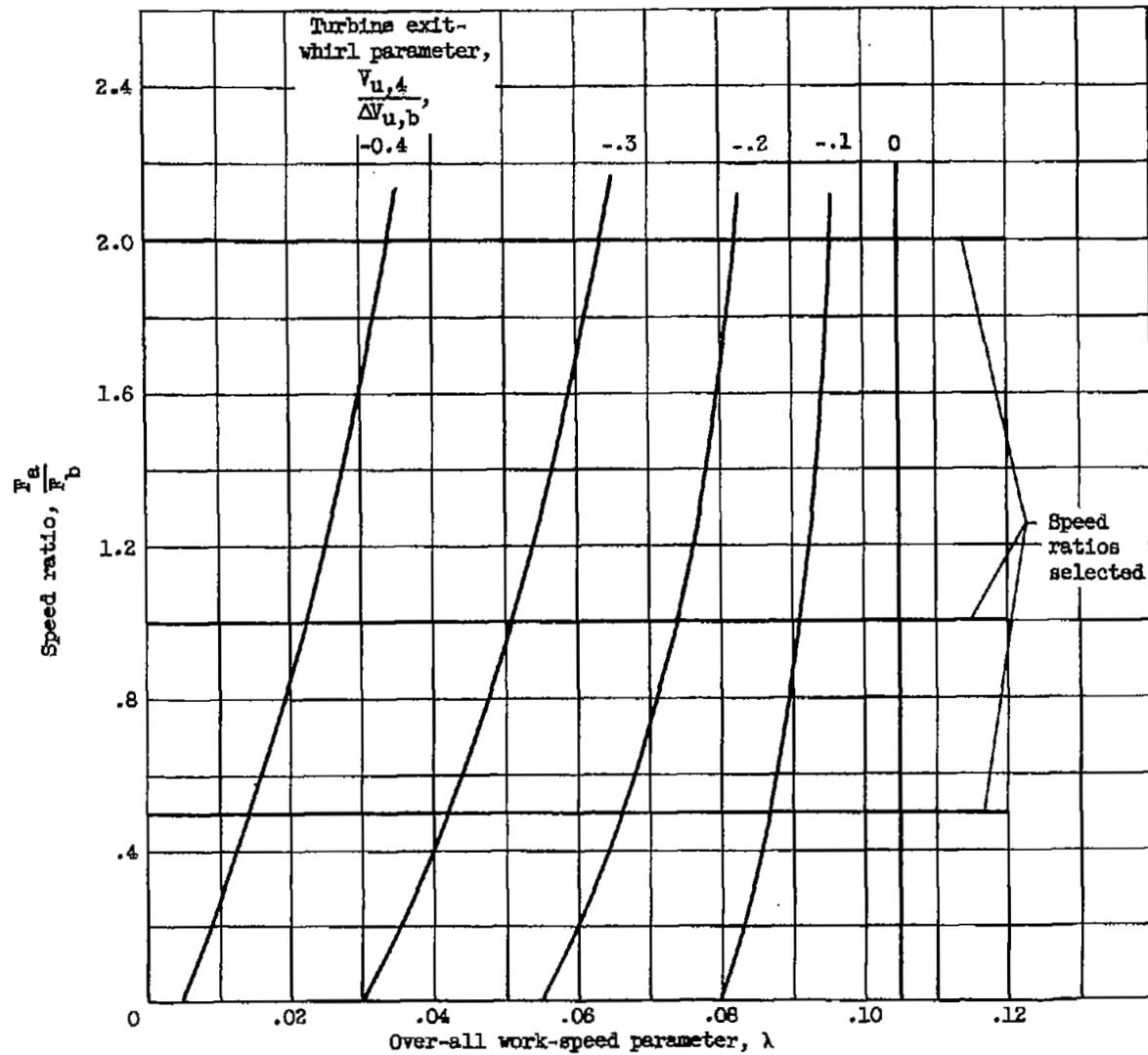
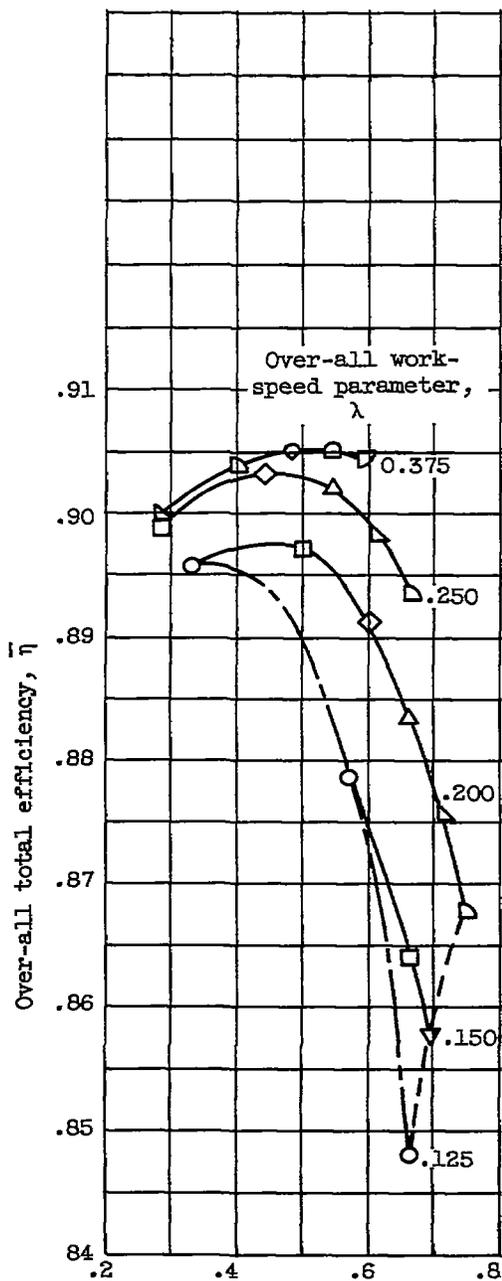


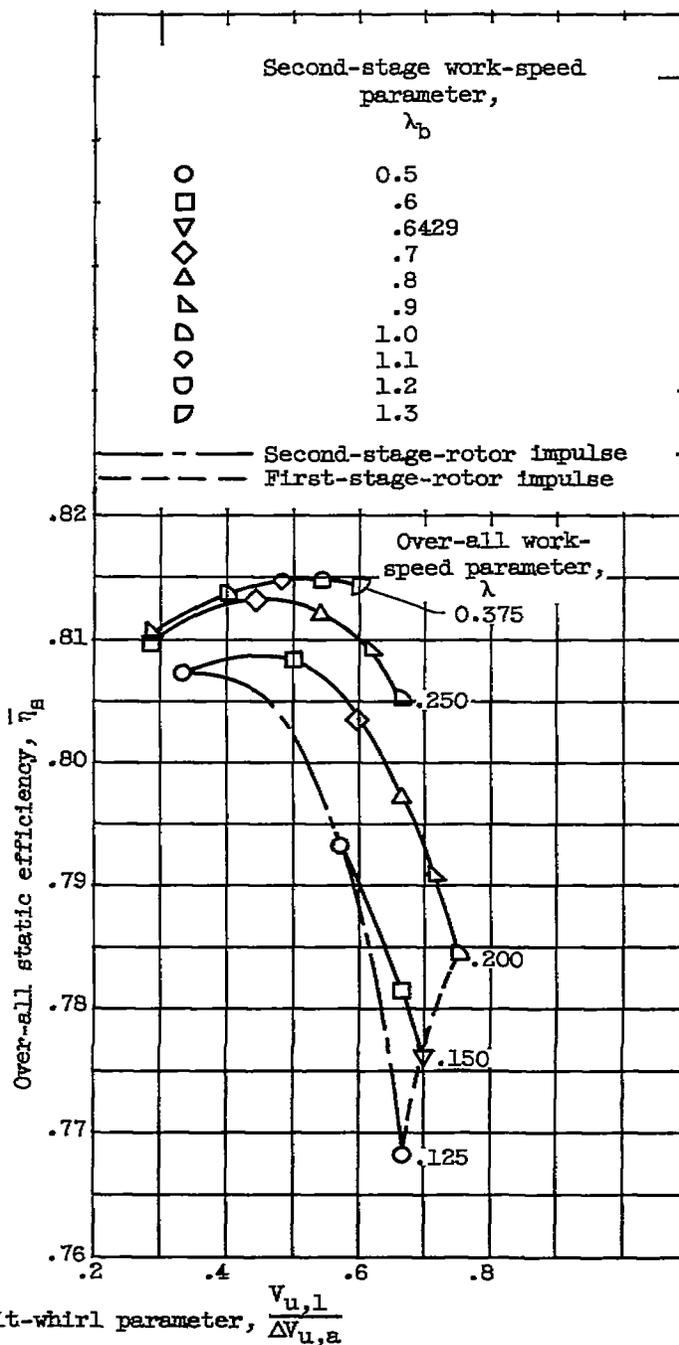
Figure 2. - Graphical description of lower limits imposed on over-all work-speed parameter.

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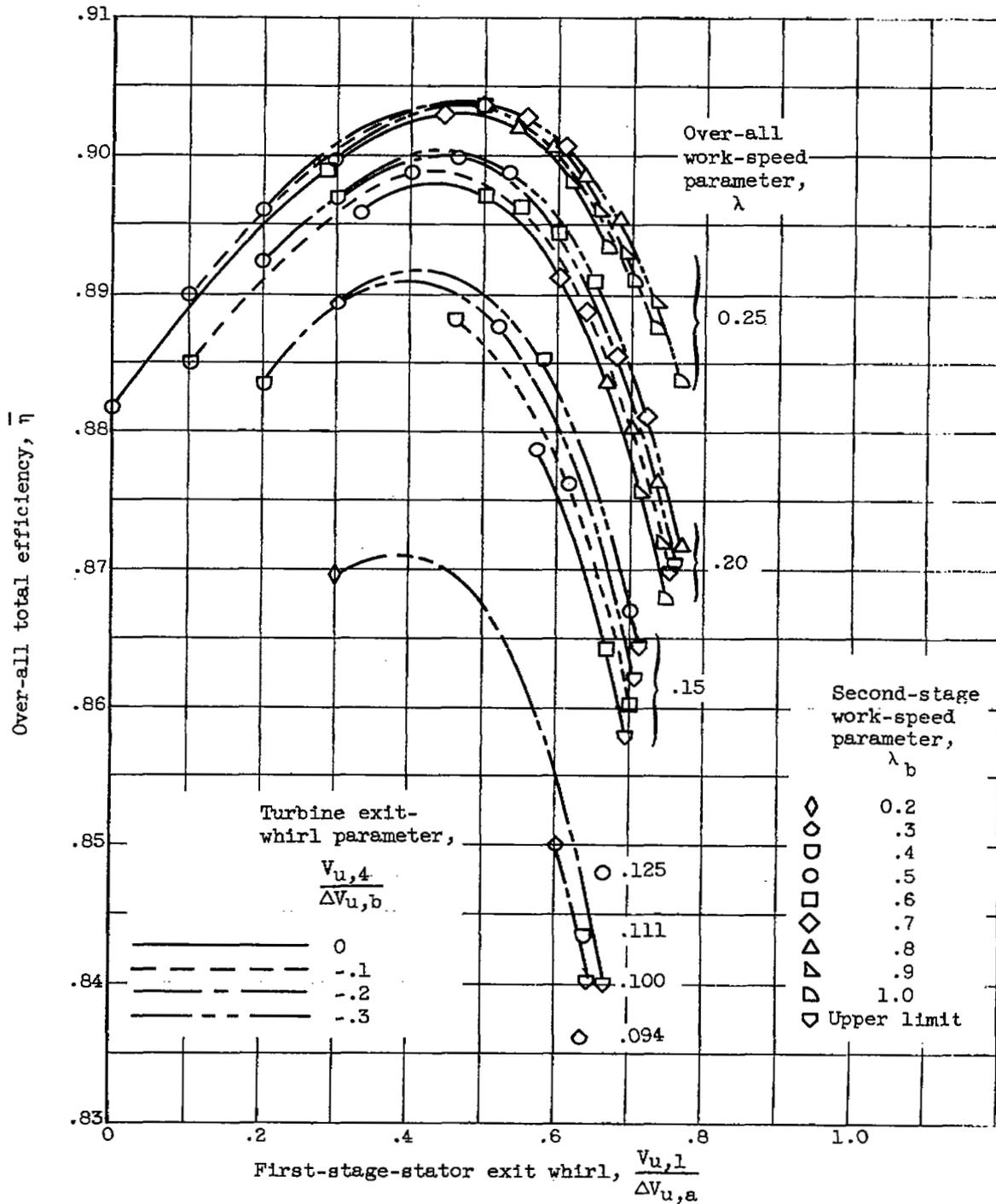


(a) Total efficiency.



(b) Static efficiency.

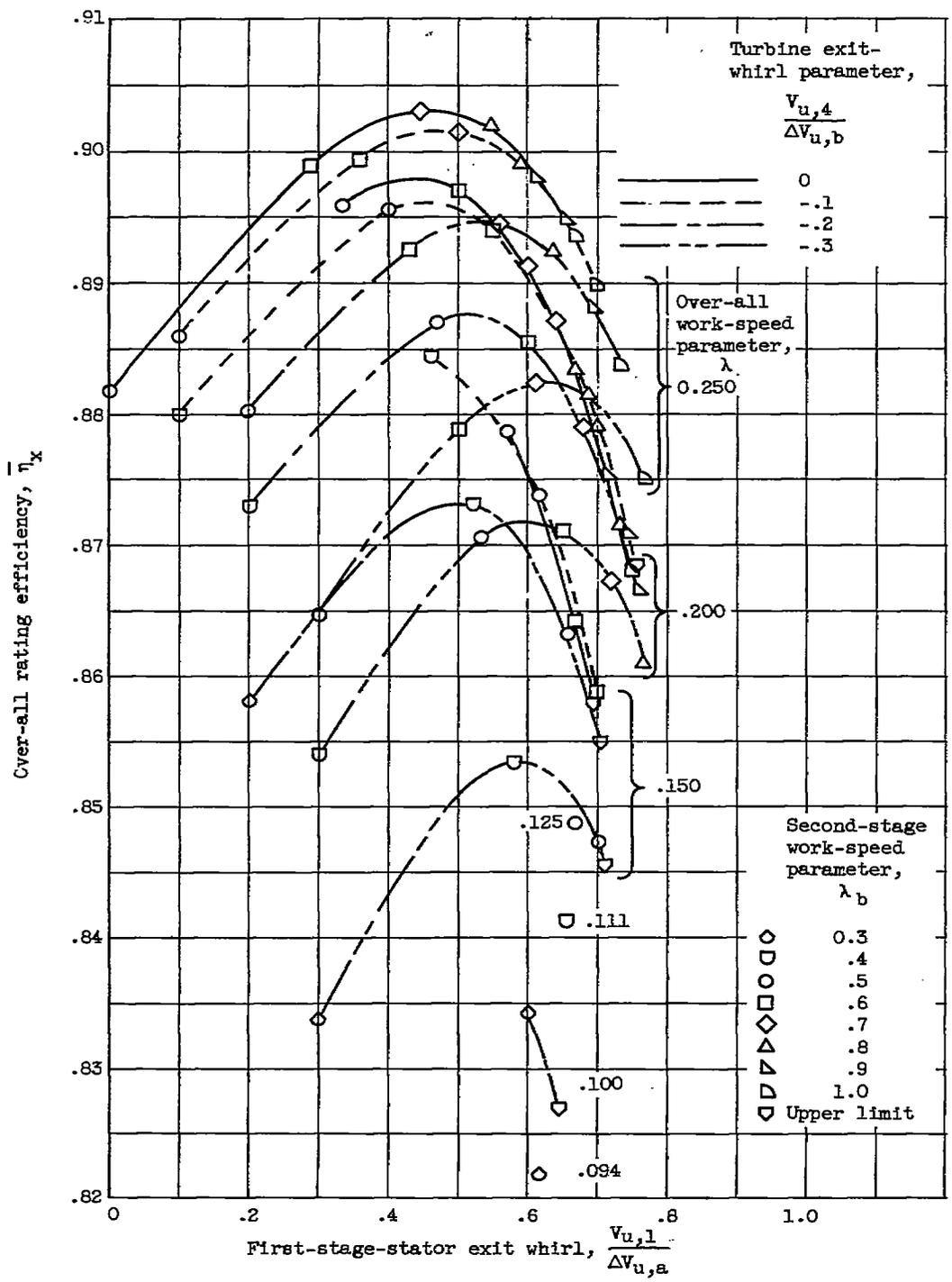
Figure 3. - Over-all efficiency characteristics; zero turbine exit-whirl parameter; equal rotor speeds, 1.0.



(a) Over-all total-efficiency characteristics.

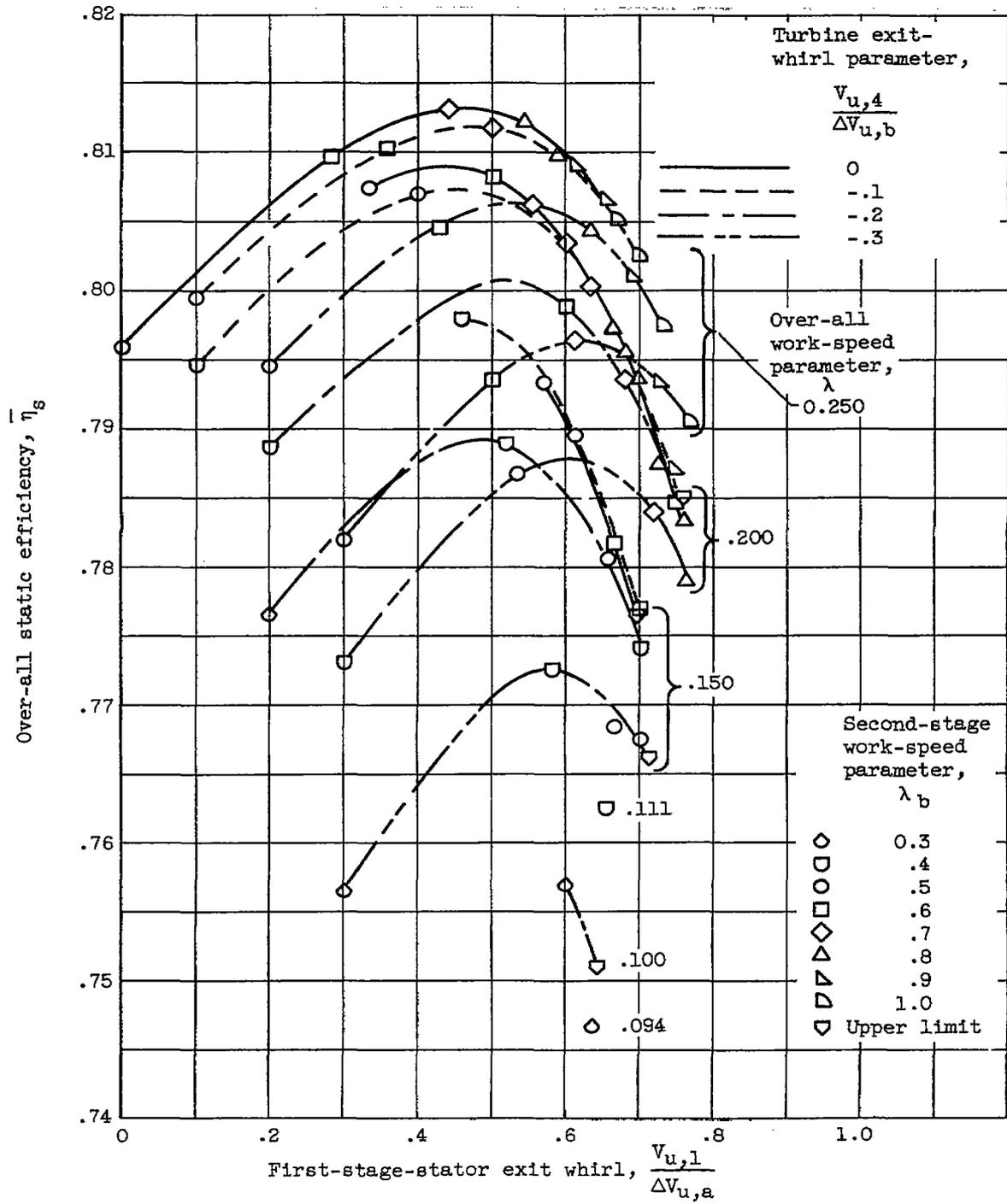
Figure 4. - Effect of turbine exit whirl for equal rotor speeds, 1.0.

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(b) Over-all rating-efficiency characteristics.

Figure 4. - Continued. Effect of turbine exit whirl for equal rotor speeds, 1.0.



(c) Over-all static-efficiency characteristics.

Figure 4. - Concluded. Effect of turbine exit whirl for equal rotor speeds, 1.0.

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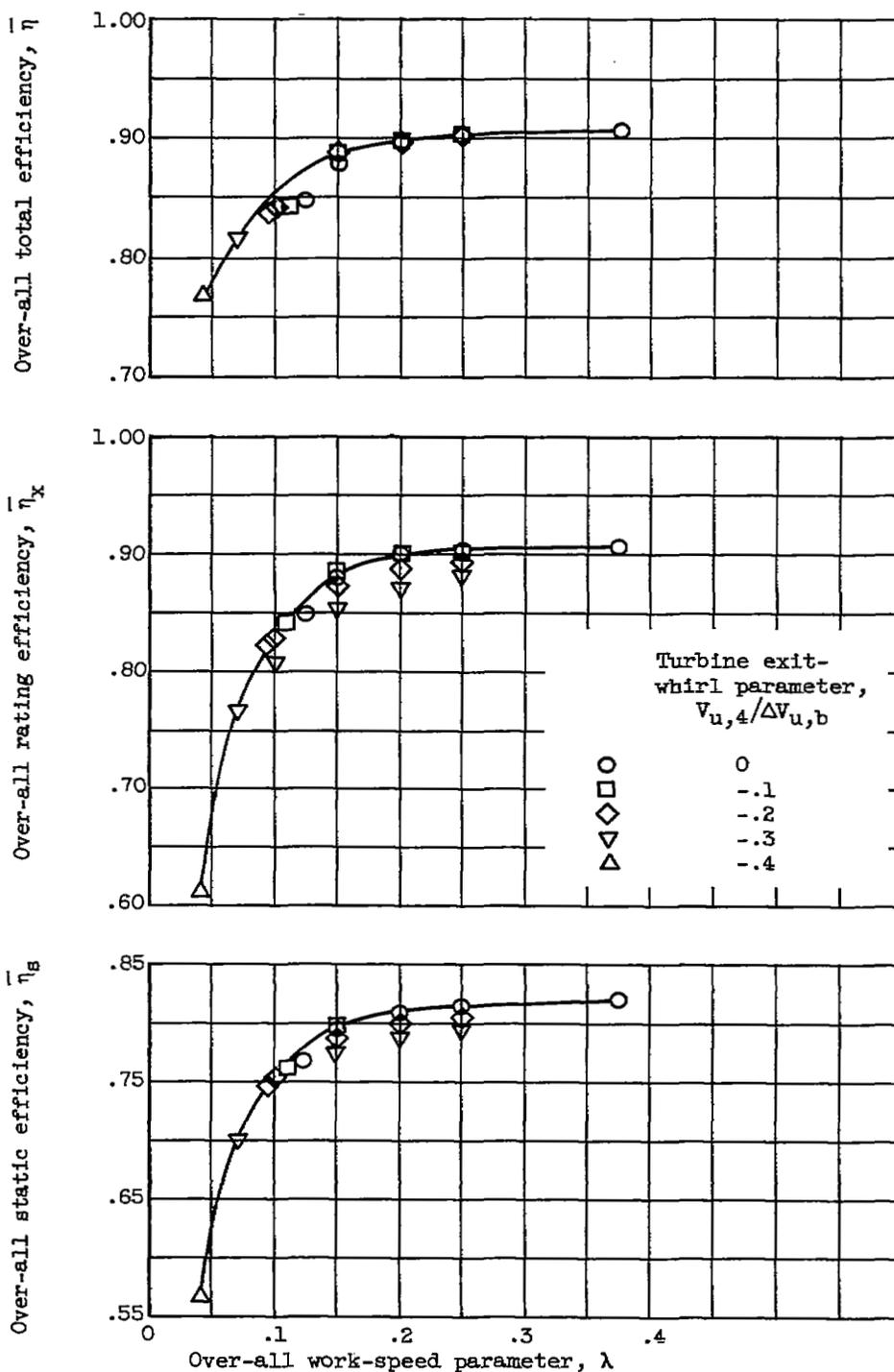


Figure 5. - Over-all maximum-efficiency characteristics with exit whirl. Speed ratio, 1.0.

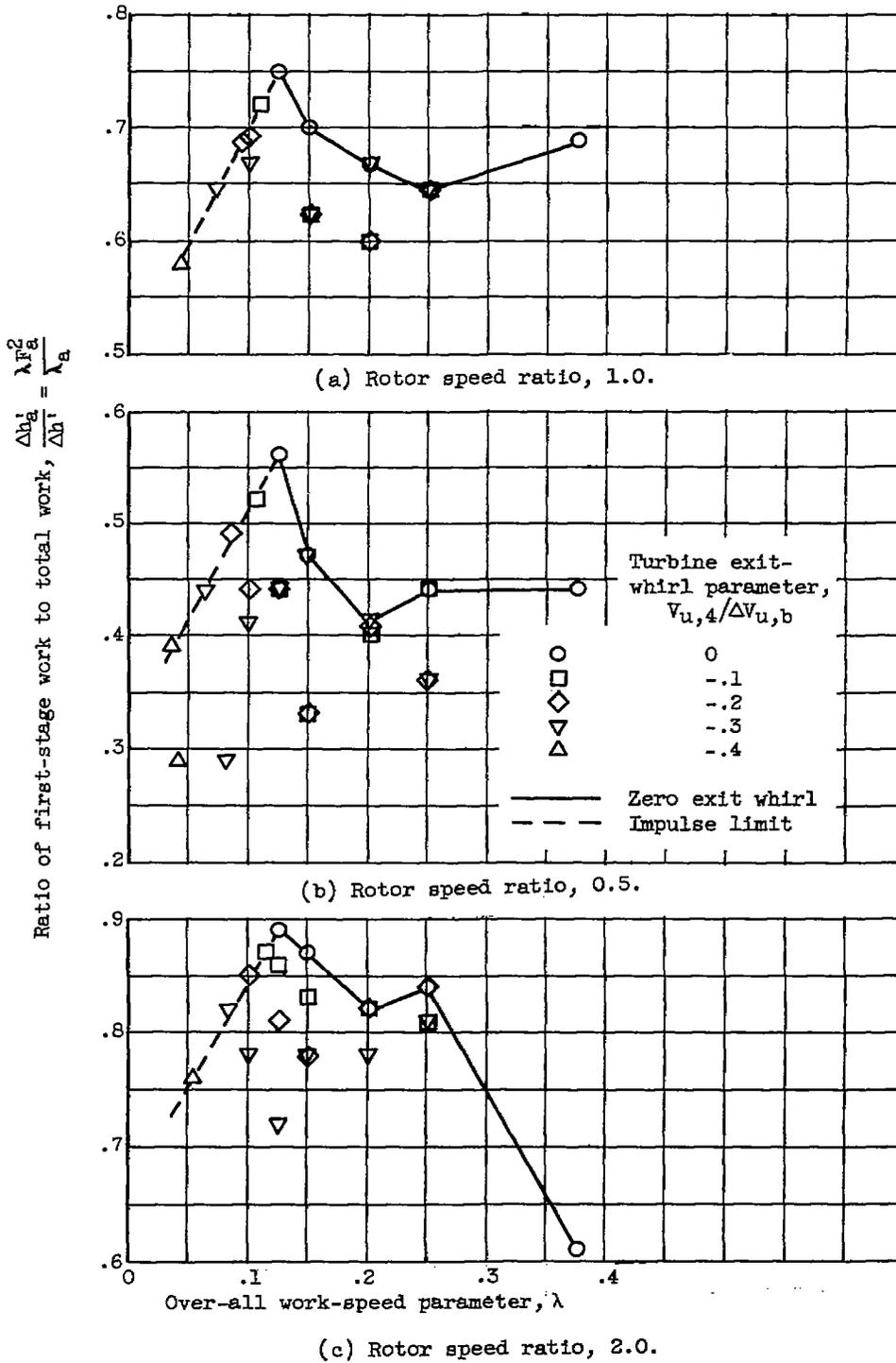


Figure 6. - Work-split characteristics with exit whirl for various rotor speed ratios.

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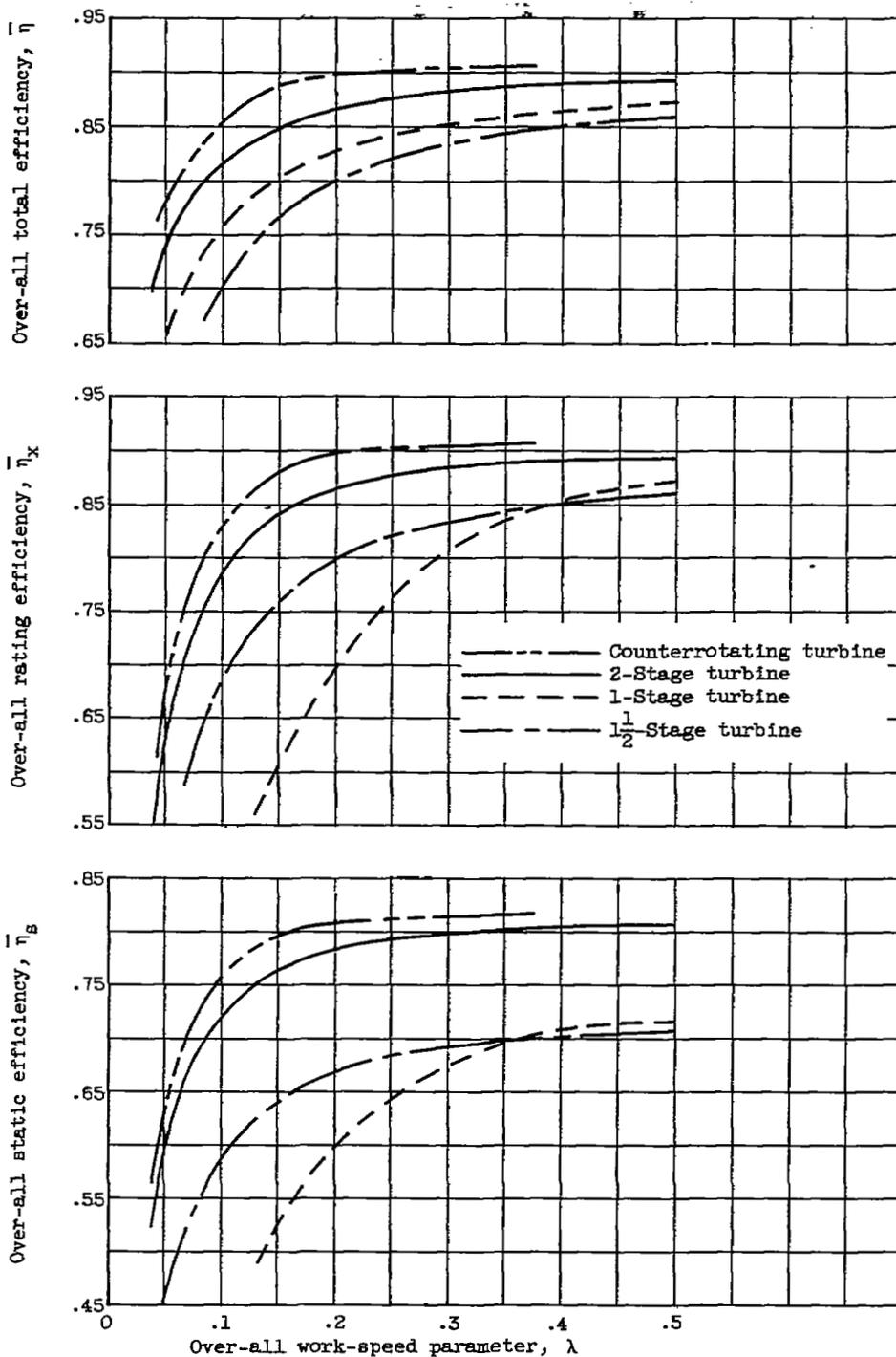
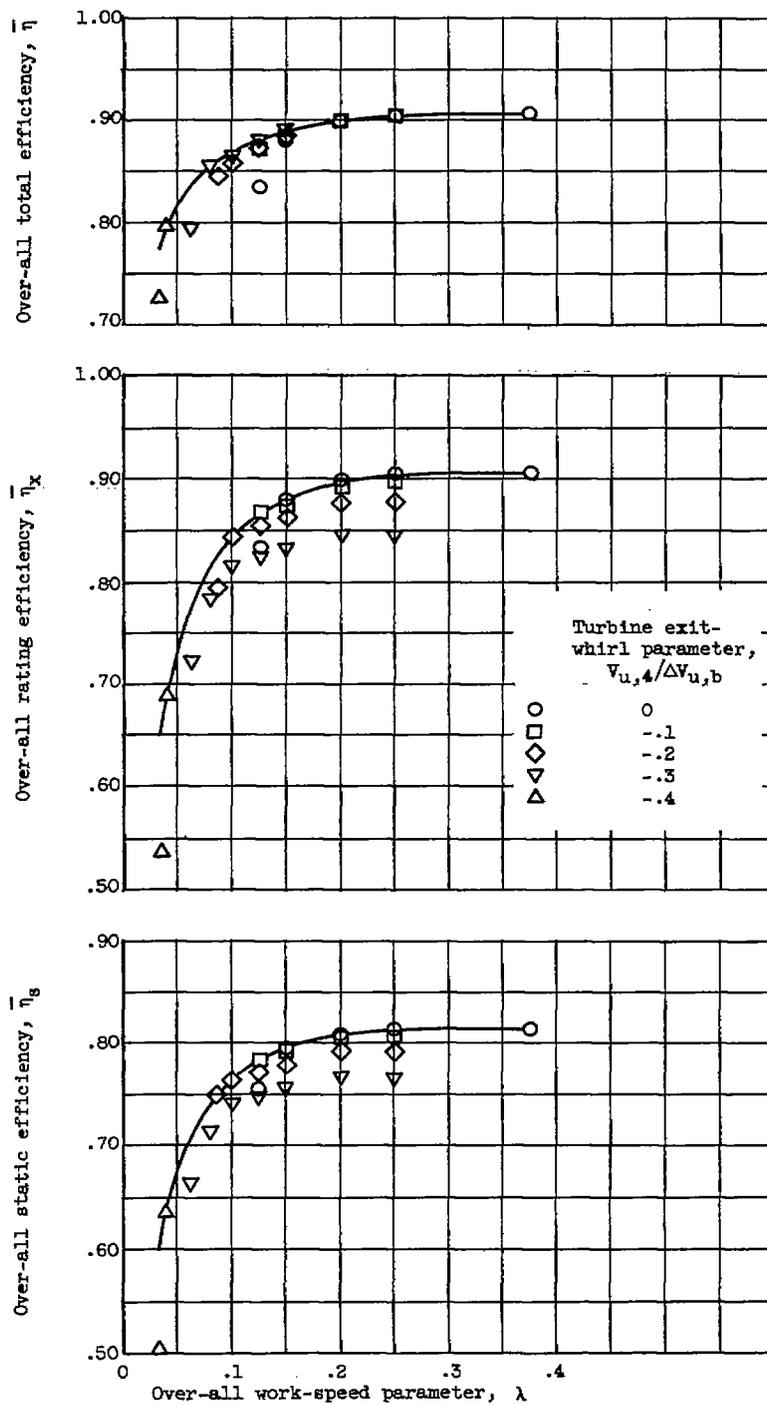


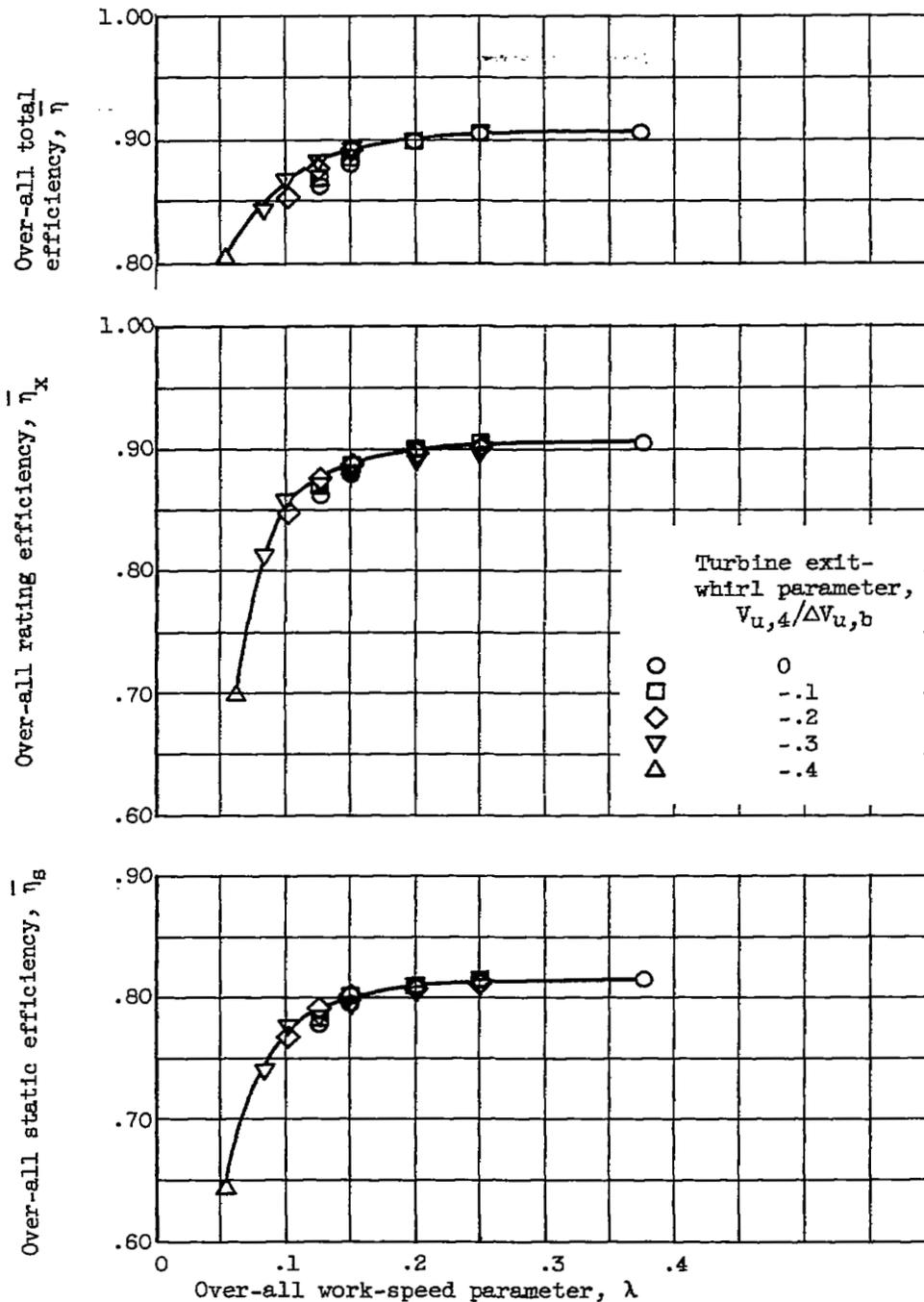
Figure 7. - Comparison of 1-, $\frac{1}{2}$, and 2-stage turbine efficiency characteristics over the range of λ from 0 to 0.5.



(a) Speed ratio, 0.5.

Figure 8. - Over-all maximum-efficiency characteristics with exit whirl.

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(b) Speed ratio, 2.0.

Figure 8. - Concluded. Over-all maximum-efficiency characteristics with exit whirl.

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